American Economic & Social Review; Vol. 3, No. 1; 2018
ISSN 2576-1269 E-ISSN 2576-1277
Impact Factor: 3.9
Published by Centre for Research on Islamic Banking & Finance and Business, USA

Testing the Validity of the Solow Growth Model: Empirical Evidence from Cross-Country Panel Data

Chigozie Nelson Nkalu[1], Richardson Kojo Edeme[1] & Queen O. Chukwuma[1]

[1]Department of Economics, University of Nigeria, Nsukka Enugu State, Nigeria

Correspondence: Chigozie Nelson Nkalu, Department of Economics, University of Nigeria, Nsukka Enugu State, Nigeria. Email: nkaluconnection@gmail.com, Phone: +2348066884155

To cite this article: Nkalu, C. N., Edeme, R. K., & Chukwuma, Q. O. (2018). Testing the Validity of the Solow Growth Model: Empirical Evidence from Cross-Country Panel Data. *American Economic & Social Review*, *3*(1), 18-22. Retrieved from http://www.cribfb.com/journal/index.php/aesr/article/view/197

Received: November 2, 2018 **Accepted:** November 8, 2018 **Online Published**: November 11, 2018

Abstract
This study seeks to test for the validity of the Solow growth model using cross-country panel data. Panel OLS analysis was adopted following an extensive review of recent and related literature with output-side of the real GDP as the dependent variable with other variables like population, capital stock and employment as the independent. However, population and capital stock are positively impacting the output with statistically significant value, while employment is not an important variable in the model even though it exhibits a negative and statistically significant effect to the output. In conclusion, the estimation result conforms the postulations of the basic Solow and augmented Solow growth model thereby validating the Solow model across-countries.

Keywords: Solow Model, Cross-Country, Panel OLS, Growth Theories, Output-side of RGDP.

JEL: A10, A20, C01, C15, C87

1. Introduction
One of the greatest challenges in economic growth and development policy is the persistent and significant differences and ranges of standards of living across countries. Different growth theories have evolved over the years starting from classical theories of growth to endogenous and neoclassical growth theories. All the growth theories strive to account for the differences in growth experiences across countries, and why the growth changes over time.

However, growth theory analyzes the relationship between the macro-dynamics and technological change and over the years, different growth models have been developed to account for and sustain the emergence and diffusion of technical change effects and the acceleration of economic growth, employment dynamics, capital stock among others (Llerena & Lorentz, 2004).

World Bank (1993) pointed out that endogenous growth models laid emphasis on international trade posits posit that due to diffusion of knowledge already existing in developed countries, high productivity growth is feasible in initially developing economies. However, the relatively underdeveloped resources in developing countries offer an opportunity to be exploited. In the newly industrialized Asian countries, faster capital growth otherwise known as accelerated growth could be accounted for by closing the gap between best and actual practice.

In basic Solow model, there has been a long run constant growth rate with stable equilibrium. However, the assumption of neoclassical analytical representation of production function comprises Inada conditions, diminution marginal return to all input, and constant returns to scale and other levels of substitution among them. In the inception of the Solow model, the technological growth and population growth rate were simply exogenous, taken for example a constant savings rate with the propensity that every economy always trades in the path of Iso-savings curve (Milton, Ricardo, & Eliezer, 2005).

Despite the novelty of Solow growth model, it has some shortcomings, one of the significant limitations of the model is its premise or assumption of a closed economy with the convergence assertion or hypothesis that presupposes a group of heterogeneous countries with no type of interrelationship. Even though it has been argued that there is no model with no untrue assumption, but this can be verifiable if the end result of such model can validate the basic and working assumption. In addition to the proposition of the Solow growth model with a closed economy assumption, Barro, Mankiw and Sala-Martin (1995) made some significant attempt in construction an open economy growth model.

Another notable shortcoming of the Solow growth model implicitly divergence of income share from capital from the output of the estimated model and national account information. That is, the unsynchronized share of income from capital and the national accounting information, however, this was the brain behind the attempt by Lucas (1988) to eliminate this problem by incorporating both physical and human (like education) resources into the concept of capital. Thirdly on the limitations of Solow growth model lies the infinitesimal low estimated convergence rate, though the convergence rate has been impacted by the modification of the Solow growth model by an open economy version of the model such as Ramsy-Cass-Koopmans model with relatively augmented convergence rate. The fourth and final shortcomings of the Solow growth model is that technological progress is the ultimate factor in determining the level of growth equilibrium of relevant variables in the model (Milton, Ricardo, & Eliezer, 2005).

Nevertheless, given the shortcomings of the Solow growth model, the model laid a strong foundation of growth theories in the economic literature with emphasis on the behavior of income growth across countries. Therefore, this study utilizes data from cross-country panel to study to verify the validity the Solow growth theories across countries as well as ascertain the magnitudes of population, employment and capital stock of growth. On that note, this study is organized as follows. Section 1 presents the introductory of dynamics of economic theories; section 2 presents a brief empirical literature review and underlying analytical underpinning of the current study. Section 3 discusses the theoretical framework with a model specification, and Section 4 presents the result and discussions with a conclusion.

2. Brief Review of Related Literature

Many researchers have attempted to test for the validity of the Solow neoclassical growth model in recent times, and notable among them is work of Mankiw, Romer, and Weil (1992). This study pointed out that with the introduction of the human capital variable in the model, the data will be well fitted with lend credence to the divergence in income disparities across countries. This study utilized new econometric methodology to provide new evidence on the behavior of income disparities across countries using time series data. Just as Romer and Weil (1992), Hoeffler (2002) used cross-country panel with OLS and fixed effect model to test for the validity of Solow growth model. The study also employed instrumental variables (IV), system and first-differenced GMM technique, the study revealed that the augmented Solow model is better explained by system GMM estimator, and thus concluded that poor growth performance of Sub-Saharan Africa is completely accounted for by augmented Solow model.

Durlauf, Kourtellos and Minkin (2001) in testing for the validity of Solow growth model using a general growth function or equation highlighted that the assumption of identical Cobb-Douglas production technology is unsatisfactory when analyzing cross-country. Related to this findings is Duffy and Papageorgiou (2000) who found alternative production function more satisfactory than the standard Cobb-Douglas production. However, Durlauf et al. (2001) also noted that strong heterogeneity is more evidenced in poorer countries in comparison with other richer countries with high tendencies of omitted variables in the nonlinear growth process. The result also revealed that higher income per capita countries is associated or linked with the highest coefficient which is evidenced in the unstable physical capital. The estimation result equally showed low values in growth rate in poor countries as a result of some omitted variables that was not captured by Solow model specification.

Murthy and Chien (1997) in the empirics of economic growth for OECD countries extended the Solow model by reexamining the augmented Solow model and adding a better measure of human capital, physical capital, and advancement in technological variables. This study showed that there is a high rate of convergence in OECD countries when optimal policies in human capital development and trade are enacted with a concomitant increase in savings. Similar to this study is Lee and Smith (1997) who employed panel data econometric analysis in investigating the empirical study of stochastic Solow model with emphasis on growth and convergence of multiple economies. However, the study showed that technical issues that are inherent in cross-sectional approach can be solved by panel analysis. Also evidenced in the study is the conclusion of heterogeneous convergence speed and inconsistent parameter in growth effects when a dynamic panel is utilized.

Nevertheless, Easterly & Levine (2001) in analyzing the stylized facts of growth models using growth accounting method found a high variation in the residuals of Solow model with relevance of efficiencies in the government policy, instrument framework vis-à-vis other structural and social infrastructures as prerequisites for cross-country accumulation of capital variation. Moreover, Ding and Knight (2009) studied the validity of the augmented Solow model in relation to economic growth in China using cross-country panel data of 146 countries. The estimation result from the GMM adopted showed that the augmented Solow growth model with structural change and human capital development impact significantly on economic growth. Laying more emphasis on the Chinese economy, the study equally revealed that rapid growth rate in China is attributable to huge investment in the physical capital with a concomitant change in employment and structural output and policy of low population growth.

Sumer (2012) tested for the joint validity of economic growth models introduced by Solow, Harrod-Domar, Barro and Romer using seemingly unrelated regression (SUR) model in Turkey with annual data ranging from 1980 to 2010. This study revealed that public expenditure is one of the major sources of economic growth in Turkey. The study validated the Barro model in Turkish economy by concluding that government intervention, public demand, and efficient economic structures stimulate economic growth towards steady-state.

3. Methodology

Solow (1956) theoretically nested the basic growth framework of which recent studies on growth models are built upon. Also, Romer, Mankiw, and Weil (1992) augmented the basic Solow growth model by introducing human capital into the Model. However, highlights of the basic and augmented Solow models are presented below.

The basic Solow growth model employed the neoclassical production function which state that per capita income growth is caused by the physical stock of capital (K), the stock of labour (L), and technological change (A) as represented in equation 1.1 below.with an exogenous growth rate of labour n as represented in equation 1.1 and 1.2 respectively.

$$Y = F(K, AL) \quad \ldots \ldots \quad 1.1$$

where: Y is output, K is capital, L is labour and A is knowledge or effective labour.

Note that the growth rate of labour n is exogenous as in equation 1.2 below:

$$\frac{\dot{L}}{L} = n \quad \ldots \ldots \quad 1.2$$

Also, the amount of national income save which equals investment is known as physical capital stock as shown in equation 1.3 below:

$$\dot{K}(t) = S(t) - \alpha K(t) \quad \ldots \ldots \quad 1.3$$

where: $\dot{K}$ (t) is the growth rate of capital stock, S(t) is the savings, K(t) is the actual capital stock, and α is the capital depreciation rate.

However, adding technological shift into production function gives equation 1.4 which shows further growth in per capita income can be stimulated in an economy that has reached steady-state through incorporating technological progress.

$$Y(t) = K^{\alpha}(t)L^{1-\alpha}(t)A(t) \quad \ldots \ldots \quad 1.4$$

where: o<α<1, A(t) captures technical shift and it is assumed to be exogenous at the growth rate of g.

Therefore, the central logarithmic equation of the basic Solow model is given in equation 1.5 as follows:

$$L_n Y(t) = \frac{\alpha}{1-\alpha} L_n S - \frac{\alpha}{1-\alpha} L_n (n + g + \alpha) + L_n A(o) + gt \quad \ldots \ldots \quad 1.5$$

The above equation 1.5 states that the depreciation rate (ɑ), the labour exogenous growth rate (g), the initial state of technology A(o) and the savings rate (s) determine the growth rate of per capita income.

Romer, Mankiw and Weil (1992) in their augmented Solow growth model modified the equation 1.5 by including the human capital into the aggregate production function as shown in equation 1.6 below:

$$Y(t) = K^{\alpha}(t)L^{1-\alpha-\beta}(t)H^{B}A(t) \quad \ldots \ldots \quad 1.6$$

While the physical capital per capita growth is shown as:

$$K(t) = S_k Y(t) - (n + g + r)K(t) \quad \ldots \ldots \quad 1.7$$

where: S_K = is the income share of physical capital production and r is the rental rate of capital. Therefore, the logarithmic transformation of the function of augmented Solow growth model is given in equation 1.8 below:

$$L_n S_k = \frac{\alpha}{1-\alpha-\beta} L_n (n+g+r) \ldots\ldots 1.8$$

Given the above theoretical framework based on basic and augmented Solow growth model, this study shall employ the following modified model to test for the validity of the Solow model using cross-country panel data:

$$L_n RGDPO_{it} = \beta_0 + \beta_1 POP_{it} + \beta_2 EMP_{it} + \beta_3 CK_{it} + \varepsilon_{it} \ldots\ldots 1.9$$

Where: RGDPO is the output-side of the real GDP, POP is the population, EMP is the employment, and CK is capital accumulation. Also, Ln is the natural logarithm and ε is the stochastic error term, while the subscript *it* represents the time series panel. The study will employ panel OLS random effect to test for the validity of the Solow growth model using cross-country annual panel data.

4. The Result, Discussion, and Conclusion

This section presents the result, analyze and discuss the output from the estimation. Before the result is presented, it is imperative here to present the summary statistics of variables of interest as specified in the empirical model (equation 1.9) above.

Table 1.The Summary Statistics of Variable of Interest

Variable	Mean	Std. Dev.	Min	Max	Observations
rgdpo overall	242850.9	902614.9	122.2258	1.32e+07	N = 6535
between	902614.9	122.2258	1.32e+07	n = 6535	
within	0	242850.9	242850.9	T = 1	
pop overall	32.16035	118.4157	.040731	1324.353	N = 6535
between	118.4157	.040731	1324.353	n = 6535	
within	0	32.16035	32.16035	T = 1	
emp overall	14.8108	59.87839	.023815	784.4269	N = 6040
between	59.96964	.023815	784.4269	n = 6021	
within	.0792873	13.32098	16.25312	T = 1.00316	
ck overall	696684.4	2744717	60.60447	4.53e+07	N = 6535
between	2744717	60.60447	4.53e+07	n = 6535	
within	0	696684.4	696684.4	T = 1	

Source: Computed by the Author Using STATA 13.0 Econometric Software

The Table 1. above is self-explanatory showing the overall variables of interest with the corresponding mean, standard deviation, minimum, maximum and observation within and within values.

Table 2. The Estimation Output of the Panel Regression

rgdpo	Coef.	Std. Err. t	P>t	[95% Conf.	Interval]
pop	1266.085	83.22248 15.21	0.000	1102.94	1429.231
emp	-847.8773	173.9049 -4.88	0.000	-1188.793	-506.9613
ck	.3058931	.000823 371.66	0.000	.3042796	.3075065
_cons	688.5399	2164.773 0.32	0.750	-3555.191	4932.271

R-Squared = 0.97 Adjusted R-Squared = 0.97 Prob>F = 0.000

Source: Computed by the Author Using STATA 13.0 Econometric Software

From the estimated output in Table 2, the result shows that all statistics/variables conform to the expectations of the Solow growth model. The population growth and capital stock are all positively impacting on the output. The employment variable is not positive and negatively affecting the output of the estimation though it is not an important variable in the model as pointed out in the theoretical framework of the Solow growth model. Also, the values of R-Squared and Adjusted R-Squared reveal relatively best fit, but not necessary while dealing with panel analysis. Consequently, despite the fact that all other variables conform to the Solow theoretical apriori expectation, with employment influencing the growth negatively, the result still validates the postulations of the Solow growth model.

References

Barro, R. J., Mankiw, G. N. and Sala-I-Martin, X. (1995). Capital Mobility in Neoclassical Models of Growth, *American Economic Review*, 85, 103 – 15.

Ding, S. and Knight, J. (2009). Can the augmented Solow Model explain China's remarkable economic growth? *A Cross-Country Panel Data Analysis. Journal of Comparative Economics,* 37, 432-452.

Duffy, J. and Papageorgiou, C. (2000). A Cross-Country Empirical Investigation of the Aggregate Production Function Specification. *Journal of Economic Growth,* 5**,** 87-120.

Durlauf, S. N., Kourtellos, A. and Minkin, A. (2001). The local Solow growth model. *European Economic Review,* 45**,** 928 – 940.

Easterly, W. and Levine, R. (2001). It is not factor Accumulation stylized facts and growth models. *Working Papers Central Bank of Chile*.

Hoeffler, A. (2002). The augmented Solow model and the African growth debate. *Oxford Bulletin of Economics and Statistics,* 64**,** 135 – 158.

Llerena, P., and Lorentz, A. (2004). Alternative Theories of Economic Growth and Co-Evaluation of Macro-Dynamics and Technological Change: A Survey. Laboratory Economics and Management (LEM), Working Paper Series.

Lucas, R. (1988). On the Mechanics of Economic Development, *Journal of Monetary Economics,* 22, 3–42.

Mankiw, N. G., Romer, D., and Weil, D. N. (1992). A Contribution to the Empirics of Economic Growth, *Quarterly Journal of Economics,* 58, 407–437.

Milton, B. F., Ricardo, G. S., and Eliezer, M. D. (2005). The Empirics of the Solow Growth Model: Long-term evidence. *Journal of Applied Economics, Vol. 8(1), 31-51*

Murthy, V. N. and Chien, I. (1997). The Empirics of Economic Growth for OECD Countries: Some New Findings. *Economics letters,* 55**,** 425 – 429.

Sumer, K. K. (2012). Testing the Validity of Economic Growth Theories with Seemingly Unrelated Regression Models: Application to Turkey in 1980-2010. *Applied Econometrics and International Development,* 12(1), 63 – 72.

Copyrights